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Auxiliary Field Meson Model at Finite Temperature and Density

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Abstract

Starting from many quark interactions, we construct a nonlinear σ-ω model at finite temperature and density. The mesons are introduced as auxiliary fields. Effective quark-meson couplings are strongly related to effective meson masses, since they are derived simultaneously from the original many quark interactions. In this model, even if the effective ω-meson mass decreases due to the partial chiral restoration, the equation of state (EOS) of nuclear matter can become soft.

1 Introduction

The ω-meson is important for the nuclear structure. It is reported that the reduction of the effective ω-meson mass makes the nuclear matter EOS stiffer. [1] However, if we require that the ω-meson mean field be proportional to the baryon density, the effective ω-nucleon coupling also becomes smaller as the effective ω-meson mass becomes smaller and the EOS of nuclear matter becomes softer. [2]

In this paper, we show that, at finite temperature and density, effective meson-quark couplings are strongly related to effective meson masses, if the meson fields are introduced as auxiliary fields which consist of quarks and anti-quarks. Consequently, if the effective ω-meson mass decreases, the effective ω-quark (or nucleon) coupling decreases and the EOS of nuclear matter becomes softer. Therefore, even if the effective ω-meson mass decreases due to the partial chiral restoration, the EOS of nuclear matter can become soft in this model.

2 Auxiliary field method for nonlinear σ-ω model

In this section, using the auxiliary field method, [3,4] we construct a nonlinear σ-ω model. (For details, see the reference [5].) We start from the many quark interactions [4,5]

$$
\begin{aligned}
\int dt\ V &= \sum_{m+n\geqq 2}\frac{1}{m!n!}\int d^4x_1\cdots d^4x_m d^4y_1\cdots d^4y_m d^4u_1\cdots d^4u_n d^4v_1\cdots d^4v_n \\
&\times\ V^{(m,n)}_{\mu_1,\cdots,\mu_n}(x_1,\cdots,x_m,y_1,\cdots,y_m,u_1\cdots u_n,v_1\cdots v_n) \\
&\times\ :\bar\psi(x_1)\psi(y_1)\cdots\bar\psi(x_m)\psi(y_m)\bar\psi(u_1)\gamma^{\mu_1}\psi(v_1)\cdots\bar\psi\gamma^{\mu_n}\psi(v_n):, \qquad (1)
\end{aligned}
$$

where ψ is the quark field. The quantum transition amplitude is given by

$$
Z_{\rm fi} = \int D\psi D\bar\psi \exp\left(i\int d^4xL\right), \qquad (2)
$$

where L is the Lagrangian density of the system. Inserting the identity

$$
\begin{aligned}
1 \quad &= \quad \int \prod_{x,y} D\Sigma_{\rm s}(x,y) D\Sigma_\mu(x,y) D\sigma(x,y) D\omega^\mu(x,y) \\
&\quad \exp\left(i \int dxdy \Sigma_{\rm s}\left\{\sigma(x,y) - \bar{\psi}(x)\psi(y)\right\}\right) \\
&\quad \times \exp\left(i \int dxdy \Sigma_\nu\left\{\bar{\psi}(x)\gamma^\nu\psi(y) - \omega^\nu(x,y)\right\}\right),
\end{aligned} \tag{3}
$$

we introduce the auxiliary meson fields $\sigma(=\bar{\psi}\psi)$ and $\omega_\mu(=\bar{\psi}\gamma_\mu\psi)$ as well as the quark self-energies $\Sigma_{\rm s}$ and Σ_μ. In this model, the expectation values of σ and ω_0 fields are proportional to the quark scalar density and the baryon density, respectively.

Integrating the quark field, we obtain by means of the mean field approximation

$$
Z_{\rm fi} = \int D\sigma D\omega_\mu \exp\left(i\Gamma[\sigma, \omega_\mu]\right), \tag{4}
$$

where Γ is the effective action and is given by

$$
\begin{aligned}
\Gamma[\sigma, \omega_\mu] \quad &= \quad W_0[\Sigma_{\rm s}[\sigma,\omega_\mu], \Sigma_\mu[\sigma,\omega_\mu]] - \sum_{m+n\geqq 2} \int V^{(m,n)}_{\mu_1\cdots\mu_n} \sigma^m \omega^{\mu_1}\cdots\omega^{\mu_n} \\
&+ \quad {\rm Tr}\left(\sigma\Sigma_{\rm s}[\sigma,\omega_\mu] - \omega_\mu\Sigma^\mu[\sigma,\omega_\mu]\right).
\end{aligned} \tag{5}
$$

The W_0 represents the quark energy and the remaining parts represent the meson potential. The quark self-energies $\Sigma_{\rm s}$ and Σ_μ are determined by the following conditions.

$$
\frac{\delta\Gamma}{\delta\sigma} \quad = \quad -\frac{\partial}{\partial\sigma}\left(\sum_{m+n\geqq 2} \int V^{(m,n)}_{\mu_1\cdots\mu_n} \sigma^m \omega^{\mu_1}\cdots\omega^{\mu_n}\right) + \Sigma_{\rm s} = 0. \tag{6}
$$

$$
\frac{\delta\Gamma}{\delta\omega^\mu} \quad = \quad -\frac{\partial}{\partial\omega^\mu}\left(\sum_{m+n\geqq 2} \int V^{(m,n)}_{\mu_1\cdots\mu_n} \sigma^m \omega^{\mu_1}\cdots\omega^{\mu_n}\right) - \Sigma_\mu = 0. \tag{7}
$$

3 Effective meson masses, effective couplings and EOS

Because of the conditions (6) and (7), the quark self-energies are strongly related to the meson potential. Therefore, at finite temperature and density, the effective meson-quark couplings are strongly related to the effective meson masses. In the uniform and rotationally invariant matter, we obtain

$$
\frac{{m^*_\sigma}^2}{{m_\sigma}^2} \equiv \frac{\partial^2\epsilon}{\partial\sigma^2} = \frac{g^*_{{\rm s}\sigma}\Pi g^*_{{\rm s}\sigma}}{m^2_\sigma} + \frac{g^*_{{\rm s}\sigma}}{g_\sigma} \qquad \text{and} \qquad \frac{{m^*_\omega}^2}{{m_\omega}^2} \equiv -\frac{\partial^2\epsilon}{\partial\omega_0^2} = -\frac{g^*_{{\rm s}\omega}\Pi g^*_{{\rm s}\omega}}{m^2_\omega} + \frac{g^*_{{\rm v}\omega}}{g_\omega}, \tag{8}
$$

where $g^*_{{\rm s}\sigma} \equiv -\frac{\partial\Sigma_{\rm s}}{\partial\sigma}$, $g^*_{{\rm s}\omega} \equiv -\frac{\partial\Sigma_{\rm s}}{\partial\omega_0}$, $g^*_{{\rm v}\omega} \equiv -\frac{\partial\Sigma_0}{\partial\omega_0}$ and Π is the polarization function. The m_σ, m_ω, g_σ and the g_ω are the σ-meson mass, the ω-meson mass, the σ-quark coupling and the ω-quark coupling at zero temperature and zero density, respectively, and ϵ is the energy density of the system. If the effects of the mixing interaction, the term including $g^*_{{\rm s}\omega}$, can be neglected, the square of the effective ω-meson mass is proportional to the effective ω-quark coupling. Therefore, the effective ω-quark coupling decreases as the effective ω-meson mass decreases.

In Fig. 1, we show the baryon density ($\rho_{\rm B}$) dependence of the binding energy of nuclear matter at zero temperature. In the calculation, we assume that $g^*_{\rm Ns\sigma} = 3g^*_{\rm s\sigma}$ and $g^*_{\rm Nv\omega} = 3g^*_{\rm v\omega}$, where $g^*_{\rm Ns\sigma}$ and $g^*_{\rm Nv\omega}$ are the effective σ-nucleon and ω-nucleon couplings, respectively. In the nonlinear model (NLM) $g^*_{\rm Nv\omega}/g_{\rm N\omega} = {m^*_\omega}^2/m_\omega^2 \sim 0.94$ at the normal density $\rho_{\rm B0}$, whereas $g^*_{\rm Nv\omega}/g_{\rm N\omega} = {m^*_\omega}^2/m_\omega^2 = 1$ in the linear model (LM). (See Fig. 2.) Although the effective ω-meson mass decreases in the NLM, the EOS in the NLM becomes much softer than that in the LM.

4 Summary

In summary, starting from the many quark interaction, we have constructed the nonlinear σ-ω mdoel. The mesons are introduced as auxiliary fields. Effective quark-meson couplings are strongly related to effective meson masses, since they are derived simultaneously from the original many quark interactions. In this model, even if the effective ω-meson mass decreases due to the partial chiral restoration, the effective ω-quark (or nucleon) coupling decreases and the EOS of nuclear matter can become soft.

References

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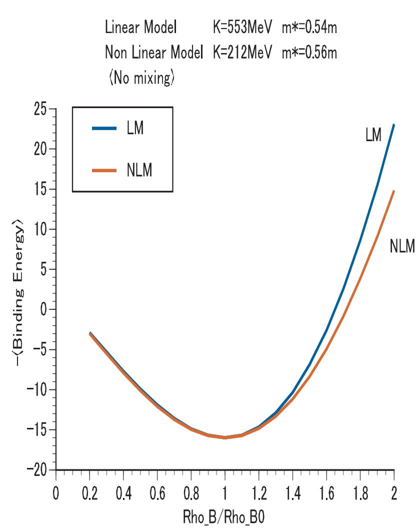

Fig.1 Binding energy (in MeV)
K: Incompressibility
m^*: Effective nucleon mass at the normal density

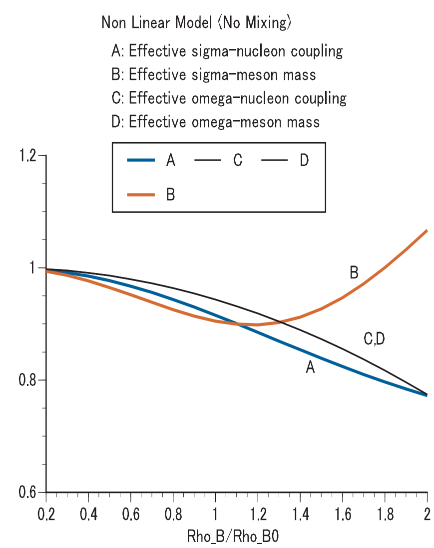

Fig.2 Squares of effective meson masses and effective meson-nucleon couplings (ratios) in NLM